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STOCHASTIC MODELLING AND OPTIMIZATION OF COMPLEX INFRASTRUCTURE SYSTEMS

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Abstract In this paper it is shown that recent progress in stochastic modelling and optimization in combination with advanced computer systems has now made it possible to improve the design and the maintenance strategies for infrastructure systems. The paper concentrates on highway networks and single large bridges. United States has perhaps the largest highway networks in the world with more than 6 million kilometers of roadway and more than 0.5 million highway bridges; see Chase, S.B. 1999. About 40% of these bridges are considered deficient and more than $50 billion is estimated needed to correct the deficiencies; see Roberts, J.E. 2001. The percentage of sub-standard bridges deemed to require urgent actions in other countries such as France (15%) and UK (20%) is also high; see Das, P.C. 1999. These figures clearly indicate that maintaining highway bridges requires more resources than usually available.

Keywords: Mathematical modelling, Finite element analysis, Corrosion cracks, Reinforced concrete.

Introduction

Obtaining and maintaining advanced infrastructure systems plays an important role in modern societies. Developed countries have in general well established infrastructure systems but most non-developed countries are characterized by having bad or no effective infrastructure systems. Therefore, in the transition from a non-developed country to a well developed country construction of effective infrastructure systems plays an important role. However, it is a fact that construction of new infrastructure systems requires great investments so a careful planning of all details in the system is essential for the effectiveness of the system from an operational but also economical point of view.

Obtaining the resources needed to establish infrastructure systems is only the first step. The next step and perhaps the most expensive step are to maintain the systems. It is recognized in most developed countries that good maintenance of infrastructure systems is in the long run the most economical way to keep the infrastructure in a satisfactory state. Effective maintenance requires however more resources than available in most countries. Therefore, careful planning of maintenance strategies is essential for all types of infrastructures.

Most of the infrastructure systems (highways, bridges, harbors, railways etc.) built in Europe in the past seventy years was designed on the basis of a general belief among engineers that the durability of the materials used could be taken for granted. Although a vast majority of infrastructure systems have performed satisfactorily during their service life, numerous instances of distress and deterioration have been observed in recent years. The causes of deterioration of e.g. reinforced concrete bridges, piers etc. are often related to durability problems of the composite material. One of the most important deterioration processes which may occur in reinforced concrete bridges is reinforcement corrosion, caused by chlorides present in de-icing salts and/or carbonation of the concrete cover zone.

1. Formulation of the Cost Optimization Problem

An infrastructure system consists of a number of structures. The objective is to minimize the cost of maintaining such a group of structures in the service life of the infrastructure. Estimation of the service life costs is a very uncertain so that a stochastic modelling is clearly needed. This can be expressed mathematically as

$$\min E[C] = \min\left(E[C_M] + E[C_U] + E[C_V]\right) \tag{1}$$

where

$E[C]$	is the expected total cost in the service life of the infrastructure
$E[C_M]$	is the expected maintenance cost in the service life of the infrastructure
$E[C_U]$	is the expected user costs e.g. traffic disruption costs due to works or restrictions on the structure
$E[C_U]$	is the expected costs due to failure of structures in the infrastructure.

For a *single* structure i in the infrastructure the expected cost can be written

$$\begin{aligned} E[C_i] &= E[C_{Mi}] + E[C_{Ui}] + E[C_{Fi}] \\ &= \sum_{t=1}^{T}\Big\{(1+\gamma)^{-1}[E[C_{Mi}(t)]P(M_{it}) \\ &\quad + E[C_{Ui}(t)]P(U_{it}) + E[C_{Fi}(t)]P(F_i(t)]\Big\} \end{aligned} \tag{2}$$

where

γ	is the discount rate (factor) e.g. 6%
$E[C_i]$	is the expected total cost for structure i
$E[C_{Mi}(t)]$	is the expected maintenance cost for structure i in year t
$E[C_{Di}(t)]$	is the expected user costs for structure i in year t
$E[C_{Fi}(t)]$	is the expected failure cost for structure i in year t
$P(M_{it})$	is the probability of the event "maintenance is necessary" for structure i
$P(D_{it})$	is the probability of the event "maintenance is necessary" for structure i
$P(F_{it})$	is the probability of the event "maintenance is necessary" for structure i
T	is the remaining service life or reference period (in years).

Let the number of structures in the considered infrastructure be m. The expected total cost for the group can then be written

$$\begin{aligned} E[C] &= \sum_{i=1}^{m}\Big\{E(C_{Mi}) + E(C_{Ui}) + E(C_{Fi})\Big\} \\ &= \sum_{i=1}^{m}\sum_{t=1}^{T}\Big\{(1+\gamma)^{-1}[E[C_{Mi}(t)]P(M_{it}) \\ &\quad + E[C_{Ui}(t)]P(U_{it}) + E[C_{Fi}(t)]P(F_i(t))]\Big\} \end{aligned} \tag{3}$$

2. Bridge Networks

Future advanced bridge management systems will be based on simple models for predicting the residual strength of structural elements. Improved stochastic modelling of the deterioration is needed to be able to formulate optimal strategies for inspection and maintenance of deteriorated reinforced concrete bridges. However, such strategies will only be useful if they are also combined with expert knowledge. It is not possible to formulate all expert experience in mathematical terms. Therefore, it is believed that future management systems will be expert systems or at least knowledge-based systems; see Thoft-Christensen, P. 1995.

Methods and computer programs for determining rational inspection and maintenance strategies for concrete bridges must be developed. The optimal decision should be based on the expected benefits and total cost of inspection, repair, maintenance and complete or partial failure of the bridge. Further, the reliability has to be acceptable during the expected lifetime.

The first major research on combining stochastic modelling, expert systems and optimal strategies for maintenance of reinforced concretes structures was sponsored by EU in 1990 to 1993. The research project is entitled *Assessment of Performance and Optimal Strategies for inspec-*

tion and Maintenance of Concrete Structures Using Reliability Based Expert systems. The results are presented in several reports and papers; see e.g. Thoft-Christensen, P 1995, 2002. The methodology used in the project is analytic with traditional numerical analysis and rather advanced stochastic modelling.

Monte Carlo simulation has been used in decades to analyze complex engineering structures in many areas, e.g. in nuclear engineering. In modelling reliability profiles for reinforced concrete bridges Monte Carlo simulation seems to be used for the first time in December 1995 in the Highways Agency project *Revision of the Bridge Assessment Rules based on Whole Life Performance: Concrete* (1995-1996, Contract: DPU 9/3/44, Project Officer: P.C. Das). The project is strongly inspired of the above-mentioned EU-project. The methodology used is presented in detail in the final project report by Thoft-Christensen, P. & Jensen, F.M. 1996.

In the Highways Agency project *Optimum Maintenance Strategies for Different Bridge Types* (1998-2000, Contract: 3/179, Project Officer: N. Haneef) the simulation approach was extended in December 1998 by Thoft-Christensen, P. 1998, 2000a to include stochastic modelling of rehabilitation distributions and preventive and essential maintenance for reinforced concrete bridges. A similar approach is used in the project by Frangopol, D.M. 2000 on steel/concrete composite bridges.

In a recent project *Preventive Maintenance Strategies for Bridge Groups* (2001-2003, Contact 3/344 (A+B), Project Officer V. Hogg) the simulation technique is extended further to modelling of condition profiles, and the interaction between reliability profiles and condition profiles for reinforced concrete bridges, and the whole life costs. The simulation results are detailed presented by Frangopol, D.M. 2003 and Thoft-Christensen, P. 2003a, 2003b.

Many authors have published a large number of reports and papers on this subject in the last decade. A number of improvements, additions and modifications are described in this literature. However, The Highways Agency projects have played a major role in this development

3. Estimation of Service Life of Infrastructures

In this paper service life assessment of infrastructures is discussed based on stochastic models and with special emphasis on deterioration of reinforced structures due to reinforcement corrosion.

The service life $T_{service}$ for a reinforced concrete structure has been the subject of discussion between engineers for several decades. Definitions related to the reliability of the structure have been proposed in recent years. Several authors; see e.g. Thoft-Christensen, P. 1997; have defined the service life as the initiation time for corrosion T_{corr} of the reinforcement. Estimating T_{corr} is a very complicated matter. An approach based on diffusion theory seems to have reached general acceptance.

The service life $T_{service}$ has later been modified so that the time Δt_{crack} from corrosion initiation to corrosion crack initiation in the concrete is included; see Thoft-Christensen, P. 2000b. The service life is then defined by $T_{service} = T_{crack} = T_{corr} + \Delta t_{crack}$. A stochastic model for Δt_{crack} may be developed on the basis of existing deterministic theories for crack initiation; see Liu, Y. & Weyers, R.E. 1998.

The service life $T_{service}$ may further be modified so that the time $\Delta t_{crack\ width}$ from corrosion crack initiation to formation of a certain (critical) crack width is included; see Thoft-Christensen, P. 2001. By this modelling it is possible to estimate the reliability of a given structure on the basis of measurements of the crack widths on the surface of the concrete structure.

Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts corroding actively. If Ficks law of diffusion can represent the rate of chloride penetration into concrete, then it can be shown that the time T_{corr} to initiation of reinforcement corrosion is

$$T^{(1)}_{service} = T_{corr} = \frac{d^2}{4D}\left((\mathrm{erf}^{-1}\left(\frac{C_{cr}-C_0}{C_i-C_0}\right)\right)^{-2} \tag{4}$$

where d is the concrete cover, D is the diffusion coefficient, C_{cr} is the critical chloride concentration at the site of the corrosion, C_0 is the equilibrium chloride concentration on the concrete surface, C_i is the initial chloride concentration in the concrete, erf^{-1} is the error function.

After corrosion initiation the rust products will initially fill the porous zone around the steel/concrete surface. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed. During this process the volume of the corrosion products at initial cracking of the concrete W_{cr} it will occupy three volumes, namely the porous zone W_{porous}, the expansion of the concrete due to rust pressure W_{expan}, and the space of the corroded steel W_{steel}. With this modelling and some minor simplifications it can then be shown that the time from corrosion imitation to crack initiation is; see Liu,Y. & Weyers, R.E. 1998

$$\begin{aligned}\Delta t_{crack} &= \frac{1}{2\times 0.383\times 10^{-3} D_{bar} i_{corr}} \\ &\times\left(\frac{\rho_{steel}}{\rho_{steel}-0.58\rho_{ust}}(W_{porous}-W_{expan})\right)^2\end{aligned} \tag{5}$$

where D_{bar} is the diameter of the reinforcement bar, i_{corr} is the annual mean corrosion rate, ρ_{steel} is the density of the steel, and ρ_{rust} is the density of the rust products.

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. Experiments (see e.g. Andrade C., Alonso, C. & Molina, F. 1993) show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time intervalΔt measured on the surface of the concrete specimen can be approximated by a linear function

$$W_{crack} = \gamma \Delta D_{bar} \tag{6}$$

where the factor γ is of the order 1.5 to 5. This linearization has been confirmed by FEM analyses; see Thoft-Christensen, P. 2003c. Let the critical crack width be $W_{critical}$ corresponding to the service life $T^{(3)}_{service}$. By setting $T^{(3)}_{service} = W_{critical}$ the following expression is obtained for $T^{(3)}_{service}$

$$T^{(3)}_{service} = \frac{W_{critical}(T_{crack})}{\gamma c_{corr} i_{corr}} + T_{crack} \tag{7}$$

$W_{crack}(T_{crack}) \approx 0$ is the initial crack width at the time T_{crack}. Using Monte Carlo simulation, the distribution functions of $T^{(1)}_{service}$, $T^{(2)}_{service}$ and $T^{(3)}_{service}$ can then for a given structure be estimated for any value of the critical crack width when stochastic distributions are known for all parameters.

4. Stochastic Modelling of Maintenance Strategies

After a structural assessment of the reliability of a reinforced concrete bridge deck at the time the problem is to decide if the bridge deck should be repaired and, if so, how and when it should be repaired. Solution of this optimization problem requires that all future inspections and repairs are taken into account. After each structural assessment the total expected benefits minus expected repair and failure costs in the residual lifetime of the bridge are maximized considering only the repair events in the residual service life of the bridge.

In order to simplify the decision modelling it is assumed that N_R repairs of the same type are performed in the residual service life $T_{service}$ of the bridge. The first repair is performed at the time T_{R_1}, and the remaining repairs are performed at equidistant times at the time interval $t_R = (T_{service} - T_{R_1})/N_R$. This decision model can be used in an adaptive way if the model is updated after an assessment (or repair) and a new optimal repair decision is made with regard to t_R. Therefore, it is mainly the time T_{R_1} of the first repair after an assessment, which is of importance. In order to decide which repair type is optimal after a structural assessment; the following optimization problem is considered for each repair technique, Thoft-Christensen, P. 1995:

$$\begin{aligned}\max_{R_R,N_R} W(T_R,N_R) &= B(T_R,N_R) - C_R(T_R,N_R) - C_F(T_R,N_R)\\ &= \text{s.t. } \beta^U(T_{service},T_R,N_R) \geq \beta^{\min} \\ &\text{or/and } T_{service}(T_R,N_R) \geq T^{\min}_{service}\end{aligned} \tag{8}$$

where the optimization variables are the expected number of repairs N_R in the residual service life and the time T_R of the first repair. W are the total expected benefits minus costs in the residual lifetime of the bridge. B is the benefit. C_R is the repair cost capitalized to the time $t = 0$ in the residual service life of the bridge. C_F are the expected failure costs capitalized to the time in the residual service life of the bridge. $T_{service}$ is the expected service life of the bridge. β^U is the updated reliability index. $\beta^{\min}$ is the minimum reliability index for the bridge (related to a critical element or to the total system). $T^{\min}_{service}$ is the minimum acceptable service life.

The benefits B play a significant role and are modelled by

$$B(T_R,N_R) = \sum_{i=[T_O]+1}^{[T_{service}]} B_i(1+r)^{T_O-T_{ref}} \frac{}{1}(1-r)^{T_i-T_O} \tag{9}$$

where $[T]$ signifies the integer part T of measured in years and B_i are the benefits in year i (time interval $[T_{i-1},T_i]$). T_i is the time from the construction of the bridge. The ith term in 10 represents the benefits from T_{i-1} to T_i. The benefits in year i are modelled by $B_i = k_0 V(T_i)$ where k_0 is a factor modelling the average benefits for one vehicle passing the bridge.

The expected repair costs C_R capitalized to the time $t = 0$ are modelled by

$$C_R(T_R,N_R) = \sum_{i=1}^{N_R} \left(1 - P_F^U(T_{R_i})\right) C_{R_=}(T_{R_o}) \frac{}{1}(1-r)^{T_{R_i}-T_O} \tag{10}$$

$P_F^U(T_R)$ is the updated probability of failure in the time interval $]T_O, T_R]$. The updating is based on a no failure event and the available inspection data at the time $_0$. The factor $(1 - P_F^U(T_{R_i})$ models the probability that the bridge has not failed at the time of repair. r is the discount rate. $C_{R0}(T_{R_i})$ is the cost of repair.

The capitalized expected costs C_F due to failure are determined by

$$C_F(T_R, N_R) = \sum_{i=1}^{N_R+1} C_F(T_{R_i}) \left(P_F^U(T_{R_i}) - P_F^U(T_{i-1_i}) \right) \frac{}{1} (1-r)^{T_{R_i}} \tag{11}$$

HERTIL where is the time of the structural assessment and is the expected service life. The ith term in (12) represents the expected failure costs in the time interval . is the cost of failure at the time .

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